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Large branes in AdS and their field theory dual

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Abstract: Recently it was suggested that a graviton in $AdS_5 \times S^5$ with a large momentum along the sphere can blow up into a spherical D-brane in S^5. In this paper we show that the same graviton can also blow up into a spherical D-brane in AdS_5 with exactly the same quantum numbers (angular momentum and energy). These branes are BPS, preserving 16 of the 32 supersymmetries. We show that there is a BPS *classical* solution for SYM on $S^3 \times R$ with exactly the same quantum numbers. The solution has non-vanishing Higgs expectation values and hence is dual to the large brane in AdS.

Keywords: D-branes, AdS-CFT Correspondance.

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Contents

1. Introduction

The holographic principle [1] is a reformulation of theories with gravity as a theory without gravity in fewer dimensions. Such a reformulation implies a one-to-one correspondence for the degrees of freedom and the observables between the bulk and the boundary. This seems unlikely at first sight because these theories live on space-times of different dimensions. The critical ingredient that makes the holographic principle work is the presence of gravity in the bulk theory. It was shown, in a number of examples, that distinct states which are indistinguishable from the boundary in the absence of gravity become distinguishable when the effects of gravity are taken into account [2]. Nonetheless, the complete account of bulk/boundary correspondence is beyond our present understanding of holography.

The most concrete realization of the holographic principle known to date is the AdS/CFT correspondence [3]. The holographic mapping of the degrees of freedom and the observables are much better understood for this class of theories. The canonical example of this correspondence is the duality between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in $3 + 1$ dimensions (boundary) and the type-IIB string theory on $AdS_5 \times S^5$ (bulk). Under this correspondence, the Kaluza-Klein modes of the supergravity fields on AdS_5 are identified with the chiral primary operators on the SYM. This mapping is justified in part by the fact that both sides assemble into short supermultiplets of the superconformal algebra. In the Hamiltonian treatment of the SYM on $R \times S^3$, these chiral primary operators can be associated to the physical states of the theory created by acting with the operator on the vacuum in the infinite past. On the AdS side, such a state corresponds to exciting the associated Kaluza-Klein mode. The energy of such an excited state (in units of the AdS radius) is the dimension of the chiral primary operator (see [4, section 3.3]). This provides a concrete identification of some of the states in the boundary and in the bulk.

In a recent paper, it was suggested on the contrary that this picture should be drastically different when the effect of angular momentum along S^5 is taken into account [5]. These authors considered Kaluza-Klein excitations carrying some angular momentum along the S^5 and considered the possibility that there exists a stable configuration of spherical branes in S^5 carrying the same quantum numbers. Although spherical branes are unstable against shrinking due to their own tensions in the trivial vacuum, there is an additional repulsive force due to the coupling to the background Ramond-Ramond field in its presence. The authors of [5] found that there indeed exists a stable spherical brane configuration.

On one hand, the observation of [5] is very intriguing. The size of the spherical brane grows with angular momentum. However, since the size of the brane can not exceed the size of the S^5, there is a bound on the allowed angular momentum for the spherical branes. This appears to offer a natural explanation for the stringy exclusion principle [6]. However, one very important puzzle is raised by the existence of such a spherical brane. There appear to be *two* states on the supergravity side corresponding to the state created by the chiral primary operators on the SYM side. This raises several questions regarding the nature of holographic principle in AdS/CFT correspondence. Which of these states should one associate with the chiral primary operators on the SYM side? More importantly, what is the SYM interpretation of the states not corresponding to the chiral primaries? One should be able to address these questions in order to resolve the holographic dictionary.

In this article, we will demonstrate that the situation is even more complicated. In addition to the stable configuration of spherical 3-branes in S^5, there is yet another stable configuration of spherical 3-brane in AdS_5 with exactly the same quantum numbers. One must therefore find the appropriate SYM interpretation to all of these brane configurations.

This paper is organized as follows. We will begin in section 2 by briefly reviewing the spherical D3-branes in the S^5. In section 3, we will construct the configuration of spherical D3-branes in AdS_5 and describe some of its properties. In section 4, we will construct a classical solution to the equation of motion of SYM which shares much of the properties of the spherical brane solution of section 3. We will conclude in section 5. Some useful formulas are collected in the appendices A, B and C.

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2. Spherical branes in S^5

Let us begin by reviewing the original argument for the existence of stable spherical brane configurations in S^5 [5]. We will work with AdS_5 in global coordinates

$$ds^2 = \frac{R^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega_3^2\right) + R^2 d\Omega_5^2\,, \tag{2.1}$$

$$C_{t\Omega_3\Omega_3\Omega_3} = TR^4\tan^4\rho\,, \tag{2.2}$$

where R is the radius of AdS_5.

Consider a graviton with angular momentum L along the S^5. In the presence of the background 5-form field strength, one might expect such a graviton to lower its own energy by "blowing up" into a spherical D3-brane along the lines of Myers' mechanism described in [7]. This can not actually happen in this case because the graviton saturates the BPS bound and its energy can not be made any smaller.[1] At best, one can expect to find a spherical brane carrying the same energy $E = P = L/R$ as the graviton.

Let us see that this is indeed the case. The lagrangian for this system is given by

$$\mathcal{L} = -TR\Omega_3 r^3\sqrt{1-\left(1-\frac{r^2}{R^2}\right)\omega^2} + \omega N\frac{r^4}{R^4}\,. \tag{2.3}$$

where

$$\omega = \frac{d\phi}{d\tau}\,, \tag{2.4}$$

and ϕ is the angular parameter along the equator of S^5. Due to the rotational invariance along the equator, the angular momentum $L = \partial\mathcal{L}/\partial\omega$ is conserved. Similarly, the conserved energy (in units of $1/R$) is

$$E = \omega L - \mathcal{L} = \sqrt{\frac{N^2 r^6}{R^6} + \frac{(L - Nr^4/R^4)^2}{1 - r^2/R^2}}\,. \tag{2.5}$$

The energy E as a function of r is illustrated in figure 1. The local minimum of E at $r = \sqrt{L/N}R$ corresponds to the stable configuration of spherical D3 brane of that radius. The fact that $\sqrt{L/N}R$ must be smaller than R places a bound $L \leq N$ on the angular momentum, which was interpreted in [5] as the manifestation of the stringy exclusion principle [8]. Looking at the form of figure 1, however, it is clear that there is another minimum at $r = 0$. This is a perfectly good solution to the equation of motion, at least classically.[2] Moreover, it is clear that this minimum exists also

[1] Even in the original context of dielectric D0-brane described in [7], classical instability for the D0-branes to blow up into a spherical D2-brane is stabilized when the gravitational back reaction of the background RR field strength is taken into account. We will elaborate further on this point in the appendix A.

[2] Due to its very small size, there will be a strong curvature correction to the DBI action.

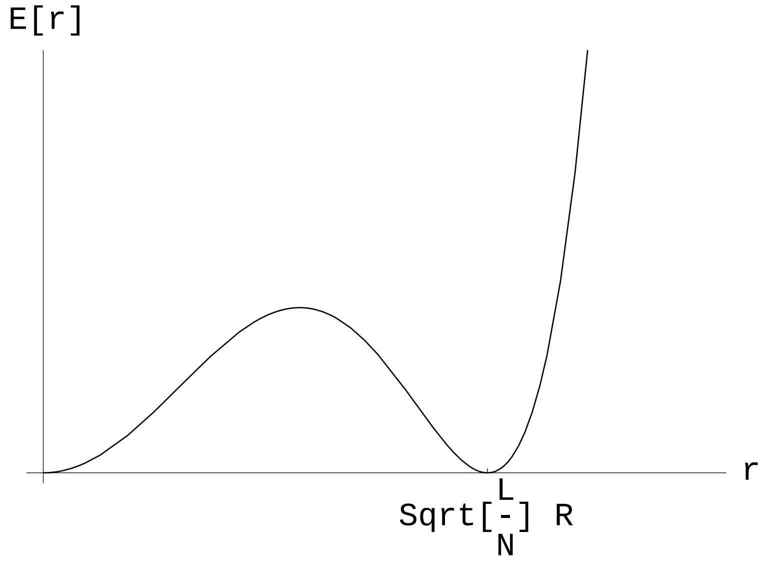

Figure 1: Energy of spherical brane as a function of its radius. The local minima of this curve corresponds to classically stable brane configurations.

for $L > N$. This raises a serious conundrum in AdS/CFT correspondence: If the minimum at $r = \sqrt{L/N}R$ is to correspond to the chiral primary operators (to match with the stringy exclusion principle), to what does the minimum at $r = 0$ correspond? To make matters worse, we will show that there is one more configuration of spherical D-brane in AdS_5 carrying the same energy and angular momentum in the following section.

3. Spherical branes in AdS_5

Existence of static spherical configuration of D3-branes in AdS_5 can be investigated along the similar lines as in the previous section. Consider embedding a spherical D3-brane wrapping the Ω_3 of the AdS_5 background (2.2). Just as in the previous section, let us consider the situation where the brane is orbiting along the equator of S^5 with angular velocity ω. The DBI action of such a brane configuration is

$$\begin{aligned} \mathcal{L} &= -\left(T\sqrt{(-g_{tt} - \omega^2 g_{\Omega_5\Omega_5}) g^3_{\Omega_3\Omega_3}} - C_{t\Omega_3\Omega_3\Omega_3}\right) \\ &= -T\Omega_3 R^4 \left(\tan^3\rho\sqrt{\sec^2\rho - \omega^2} - \tan^4\rho\right). \end{aligned} \tag{3.1}$$

Just as in the previous section, the angular momentum $L = \partial\mathcal{L}/\partial\omega$ is a conserved quantity, and the canonical energy takes the form

$$E = N\left(\sec\rho\sqrt{\frac{L^2}{N^2} + \tan^6\rho} - \tan^4\rho\right), \tag{3.2}$$

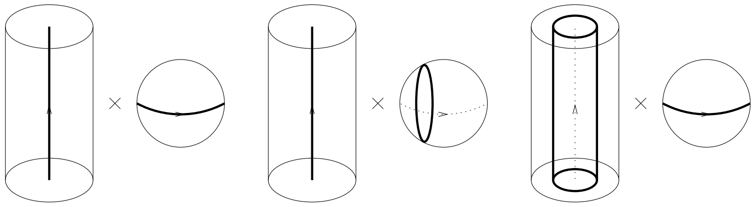

Figure 2: **I** collapsed spherical D3-brane of zero size, **II** spherical D3-brane embedded in S^5, and **III** spherical D3-brane embedded in AdS_5. These states are degenerate in energy and angular momentum quantum numbers.

where we have used the fact that $T\Omega_3 R^4 = N$. This function has essentially the same form as what is illustrated in figure 1. There are local minima at $\tan\rho = 0$ and $\tan\rho = \sqrt{L/N}$ where E takes the value (in units of $1/R$)

$$E = L\,. \tag{3.3}$$

This establishes the fact that there exists a stable configuration of spherical D3-brane embedded in the AdS_5. These brane configurations, as well as the brane configurations described in the previous section, are illustrated in figure 2.

Several comments are in order regarding the spherical brane configuration in AdS_5.

- The spherical brane in AdS_5 couples electrically to the background Ramond-Ramond field and should be thought of as a dielectric brane. The spherical brane in S^5 couples magnetically and should be thought of as a dimagnetic brane.

- There are two solutions, one at $\tan\rho = \sqrt{L/N}$ and the other at $\tan\rho = 0$, just as in the previous section. All of these brane configuration preserve the same 16 of the 32 supersymmetries of type-IIB theory on $AdS_5 \times S^5$. At first sight this is natural for they saturate the BPS bound. Nonetheless, this is a very non-trivial statement since different patches of the brane world volume are oriented in different directions. The details are explained in appendix B.

- There is an instanton solution describing the tunneling between these two minima, given by

$$\tau = \tau_0 \pm \frac{1}{2}\log\left(\frac{\sin^2\rho}{L/N - \tan^2\rho}\right), \tag{3.4}$$

whose action evaluates to

$$S = \frac{N}{2}\left(\frac{L}{N} - \log\left(1 + \frac{L}{N}\right)\right). \tag{3.5}$$

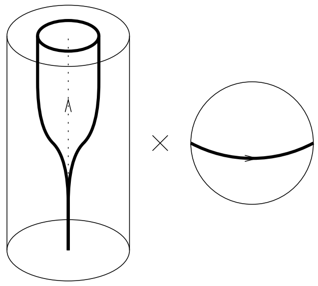

Figure 3: Instanton configuration describing the tunneling between configurations **I** and **III** of figure 2.

The form of this instanton solution[3] is illustrated in figure 3. To fully appreciate the effect of these instanton solutions, as well as the ones on the sphere, one must take the fermion zero modes into account. It turns out that all of these instantons are exactly 1/4 BPS, preserving 8 of the 32 supersymmetries, as will be explained in detail in appendix B. The supersymmetries broken by the instantons will give rise to fermionic zero modes which will suppress the mixing between the two minima via the tunneling effects.

- All of the solutions $\tan\rho = \sqrt{L/N}$, $\tan\rho = 0$, $r = \sqrt{L/N}R$, and $r = 0$ have the same energy and angular momentum quantum numbers.

- Since $\tan\rho$ is not bounded, L can be much larger than N and hence there is no apparent connection between spherical branes in AdS_5 and the stringy exclusion principle.

- These configurations are special case of the "large brane" configuration discussed in related contexts in [6, 9, 10, 11]. In general, these large brane configurations are time dependent solutions corresponding to the vacuum decay and other related phenomena. When the effect of both angular momentum and the Ramond-Ramond field is taken into account, we find a novel stationary configuration of these large branes.

The mere existence of these brane configuration raises an important question: How does one distinguish between these states from the viewpoint of the boundary theory? Unfortunately, we are unable to offer a complete resolution to this problem.

[3]Similar solution describing tunneling between spherical brane in S^5 and the point-like brane also exists

$$\tau = \tau_0 \pm \frac{1}{2}\log\left(\frac{L}{N}\frac{R^2}{r^2} - 1\right) \tag{3.6}$$

whose action evaluates to

$$S = -\frac{N}{2}\left(\frac{L}{N} + \log\left(1 - \frac{L}{N}\right)\right). \tag{3.7}$$

One very concrete and interesting observation that we discuss in the next section is that the spherical branes in AdS_5 (as opposed to the spherical branes in S^5 and the point-like brane) turns out to have a concrete interpretation as a *classical* solution from the field theory point of view.

4. Spherical branes in AdS_5 as classical solutions of SYM

In this section, we will describe a solution to the classical equation of motion of the SYM which is dual of the spherical branes in AdS.

Configuration of spherical branes in AdS_5 (illustrated in figure 2.**III**) is such that the flux of RR 5-form in the interior of the spherical D3-brane is less by one unit compared to the exterior. In light of the UV/IR relation of the AdS/CFT correspondence, this suggests that the gauge symmetry is broken from SU(N) to SU($N-1$) $\times$ U(1) at low energies. Therefore we should look for a classical configuration involving Higgs expectation values.

Since the D3-branes do not act as a source for the dilaton and the axion, the supergravity back reaction of the spherical D3-branes is trivial in the dilaton/axion sector. Trivial dilaton/axion background corresponds to trivial F^2 and $F\tilde{F}$ expectation values. The field theory counterpart of the spherical brane is therefore not likely to involve the gauge fields. Furthermore, the fact that the energy (3.3) of the solution we are after does not depend on the coupling constant suggests that the commutator term in the action of the SYM should not play any role. We are therefore left with the Abelian part of the action of the six scalar fields ϕ_i, $i = 1, \ldots, 6$.

Theories on $S^3 \times R$ contain an additional term in the action coming from the positive curvature of S^3. In n dimensions this term is fixed by the conformal invariance to be

$$S = -\frac{1}{2g_{YM}^2} \int d^n x \left((\partial\phi_1)^2 + (\partial\phi_2)^2 + \frac{(n-2)}{4(n-1)} \tilde{R} (\phi_1^2 + \phi_2^2) \right), \tag{4.1}$$

where ϕ_1 and ϕ_2 are the two scalars we focus on and $\tilde{R}$ is the Ricci curvature which is related to the radius R of AdS_{n+1} by

$$\tilde{R} = \frac{(n-1)(n-2)}{R^2}\,. \tag{4.2}$$

Setting $n = 4$, the action becomes

$$S = \frac{R^3 \Omega_3}{2g_{YM}^2} \int dt \, \left(\dot{\phi}_1^2 + \dot{\phi}_2^2 - \frac{1}{R^2} (\phi_1^2 + \phi_2^2) \right). \tag{4.3}$$

Reparameterizing the fields according to

$$\phi_1 = \sqrt{\frac{g_{YM}^2 N}{R^2 \Omega_3}} \eta \cos\theta\,, \qquad \phi_2 = \sqrt{\frac{g_{YM}^2 N}{R^2 \Omega_3}} \eta \sin\theta\,, \tag{4.4}$$

gives

$$S = \frac{NR}{2} \int dt \, \left(\dot{\eta}^2 + \eta^2 \dot{\theta}^2 - \frac{\eta^2}{R^2} \right) . \tag{4.5}$$

Now, consider the ansatz

$$\eta = \text{const.} \, , \qquad \theta = \omega t \, . \tag{4.6}$$

The angular momentum $L = dS/d\omega$ is conserved, and the conserved energy, in units of $1/R$ (see eq. (2.1)), is

$$E = L\omega - \mathcal{L} = \left(\frac{L^2}{2N\eta^2} + \frac{N\eta^2}{2} \right) , \tag{4.7}$$

which is minimized at

$$\eta = \sqrt{\frac{L}{N}} \, . \tag{4.8}$$

This constitutes a solution to the equation of motion of the field theory (4.1). The energy associated with this solution is

$$E = L \, . \tag{4.9}$$

To properly account for the $\mathrm{SU}(N)$ field content of the SYM, simply parameterize ϕ_1 and ϕ_2 according to

$$\phi_1 = \sqrt{\frac{g_{YM}^2 N}{R^2 \Omega_3}} \hat{\eta} \cos\theta \, , \qquad \phi_2 = \sqrt{\frac{g_{YM}^2 N}{R^2 \Omega_3}} \hat{\eta} \sin\theta \, , \tag{4.10}$$

where $\hat{\eta}$ is a traceless diagonal $N \times N$ matrix

$$\hat{\eta} = \sqrt{\frac{N-1}{N}} \begin{pmatrix} \eta & & & \\ & -\frac{\eta}{N-1} & & \\ & & \ddots & \\ & & & -\frac{\eta}{N-1} \end{pmatrix} . \tag{4.11}$$

To leading order in $1/N$, all but the first diagonal element can be ignored and the analysis reduces to treating $\phi_{1,2}$ as an ordinary scalar field. The subleading $1/N$ correction can be thought of as the back reaction of the spherical brane to the background geometry. Taking the full matrix structure of $\phi_{1,2}$ into account does not affect (4.7)–(4.9) for they commute.

Let us make some comments regarding this solution

- The energy of the classical solution (4.9) is precisely the energy of the spherical brane in AdS_5 found in equation (3.3).
- The magnitude of the scalar expectation value (4.8) is the same as the SUGRA result if one uses the original UV/IR relation of Maldacena [3] and not the ones of [12]. This is expected for we are dealing with Higgs expectation values and not gravitational waves as the probes in the bulk.

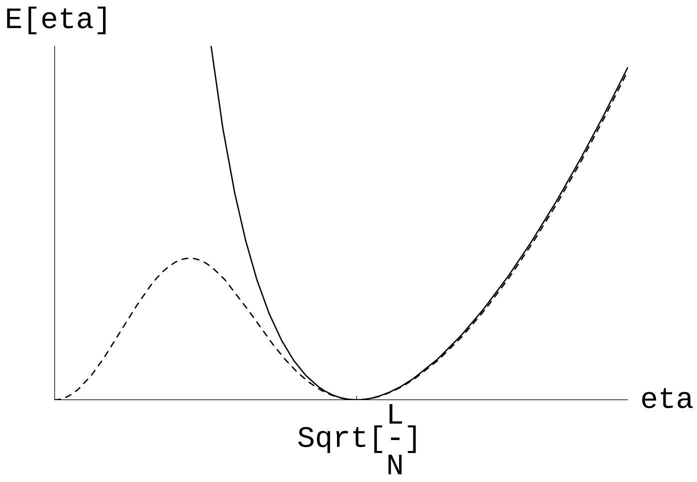

Figure 4: Solid line is energy as a function of η of the classical solution (4.7) applicable for small λ. The dotted line is the same function for brane probe action (4.12) applicable for large λ.

- The classical solution is invariant under half of the supersymmetries. This can be verified easily by acting on the solution with the supersymmetry transformation rules given in [13]. (Strictly speaking, we have only checked that the solution is invariant with respect to 8 out of 16 Poincaré supersymmetries.)

The fact that the classical solution of the SYM shares many properties in common with the spherical brane configuration in AdS_5 is a good indication that the former is the field theory realization of the latter. There are some subtle differences, however. The potential (4.7) is the field theory counterpart of (3.2). To be more precise, (3.2) is the effective action for the spontaneously broken U(1) at large λ after integrating out the massive W-bosons. Equation (4.7) can simply be thought of as the small λ limit of the same quantity. To facilitate the comparison, let us re-express (3.2) in terms of $\eta = \tan\rho$

$$E = N\left(\sqrt{1+\eta^2}\sqrt{\frac{L^2}{N^2}+\eta^6}-\eta^4\right). \tag{4.12}$$

Potentials (4.7) and (4.12) differ from each other in one very important sense. (See figure 4 for an illustration.) The potential at strong coupling (3.2) has two minima, one at $\eta = 0$ and the other at $\eta = \sqrt{L/N}$. At small coupling, (4.7) has only one minima, at $\eta = \sqrt{L/N}$.

What happened? What we have found is an argument based on duality that the minima at $\eta = 0$ is lifted by $1/\lambda$ corrections. When $\lambda \ll 1$, semi-classical description of the SYM becomes reliable, but the configuration at $\eta = 0$ simply does not exist as a solution of the classical equation of motion. It would be very interesting to understand the status of $\eta = 0$ solution when the quantum effects on the SYM side is taken into account. Studying the quantum correction to (4.7) perturbatively should teach us a lot about this issue.

Unlike the solution at $\eta = 0$, the solution at $\eta = \sqrt{L/N}$ is a robust result. This can be seen in the following way. For large values of L/N, the spherical brane will grow to have size much greater than the radius of AdS_5. In [9], Seiberg and Witten showed that the DBI+CS action of the n-brane in AdS_{n+1} has the following form for $n > 2$ near the boundary of the AdS (see eq. (3.17) of that paper)

$$S \sim \int \sqrt{g} \left((\partial\phi)^2 + \frac{n-2}{4(n-1)} \phi^2 \tilde{R} + \mathcal{O}(\phi^{\frac{2(n-4)}{n-2}}) \right) . \tag{4.13}$$

The form of this action is dictated by the fact that the extension of the metric on the boundary of AdS to the bulk is unique in the neighborhood of the boundary [14]. The leading term in large ϕ of (4.13) exactly matches the field theory action (4.1).

5. Conclusions

The main goal of this paper is to point out an important subtlety in our current understanding of holography and AdS/CFT correspondence. In AdS/CFT, there is a natural one-to-one correspondence between the chiral primary operators of the boundary theory and the Kaluza-Klein excitations on the bulk. However, there exists a configuration of spherical D-branes embedded in the S^5 *in addition* to the Kaluza-Klein excitations, carrying the same quantum numbers as was demonstrated in [5]. In this article, we demonstrated that there is *yet another* configuration of spherical D-branes, embedded in the AdS_5, again with the same quantum numbers. The full understanding of holographic principle will require that one understands how each of these spherical branes are realized on the field theory side.

The spherical branes in AdS are much like the long strings [6, 9, 11] in $AdS_3 \times S^3$ (see also appendix C). The long strings live on the boundary of AdS_3, and gives rise to new class of operators of the CFT. The spherical branes in higher AdS appears to play a slightly different role. These branes do not live at the boundary but at some definite radius in the bulk. Unlike the long strings, these branes are completely degenerate in angular momentum and energy with the Kaluza-Klein excitations.

We have not resolved the problem of identifying and distinguishing all of the brane configuration from the field theory side. To partially address this problem, we described a classical solution to the equation of motion of the SYM which shares many of the properties of the spherical branes in AdS_5. The collapsed brane configurations and the spherical branes in S^5 do not appear to correspond to a classical solution in a similar manner. Does this also imply that the $r = 0$ solution of [5] is also lifted? This depends on whether the $r = 0$ solution and the $\rho = 0$ solution can be identified as the same physical state. This is a tricky question because there is a large degeneracy of states that look like figure 2.**I** especially when multi-particle states are taken into account. More detailed understanding of the holographic map is needed to resolve this issue. Even if the $\rho = 0$ state and the $r = 0$ state turns out not to be the same physical state, the fact that the $\rho = 0$ solution was lifted by $1/\lambda$ correction is a strong indication that the $r = 0$ solution is also lifted.

From the point of view of semi-classical SYM, BPS classical solution is a coherent state of many quanta of chiral excitations. If the identification of spherical branes in S^5 with the states created by the chiral primary operators turns out to be correct, this suggests that the spherical brane in AdS_5 is a coherent state of spherical branes in S^5. It would be very interesting to understand this point better.

In order to proceed further, it appears to be necessary to properly address either the quantum correction of the SYM side or the curvature correction of the supergravity side. This is clearly a non-trivial challenge. The spherical brane configurations in supergravity do exist, and their existence is a prediction about strongly coupled gauge theory via the AdS/CFT correspondence. Learning to resolve these redundancies should teach us a lot about the dynamical aspects of quantum field theories, as well as the holographic principle.

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A. Classical stability and the gravitational back reaction

The general feature that branes of spherical geometry can exist stably in a background of some anti-symmetric tensor field strength is quite similar to the mechanism of dielectric branes discussed by Myers [7], but there is one critical difference. In AdS, the $r = 0$ solution is classically stable. In Myers' analysis, $r = 0$ solution is classically unstable. In this appendix, we will explain that even in Myers' example, $r = 0$ solution is classically stable when the effect of gravitational back reaction of the stress-energy of the background Ramond-Ramond field strength is taken into account.

Myers' analysis assumes a flat space-time in a background of constant RR 4-form field strength, giving rise to a potential of the form (see [7, equation (87)].)

$$E(r) = T_2 \left(\sqrt{\frac{\alpha'^2 N^2}{4} + r^4} - F r^3 \right) \approx T_2 \left(\frac{N\alpha'}{2} - F r^3 + \frac{r^4}{N\alpha'} \right). \tag{A.1}$$

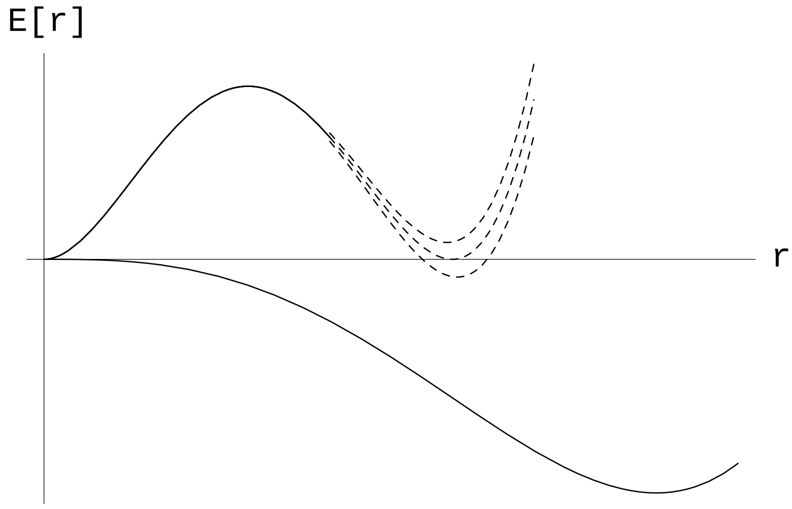

Figure 5: Energy of spherical brane in the background Ramond-Ramond field neglecting the back reaction (A.1) and including the back reaction (A.6). These curves are only valid locally near $r = 0$. When the back reaction is taken into account, $r = 0$ is classically stable. However, it does not determine if this point is a global minimum or not.

The relevant energetic consideration comes from the r^3 term which has a negative coefficient, and the r^4 term which has a positive coefficient. Since the leading small r effect is negative, there is a classical instability (see figure 5).

The essential difference between Myers' flat space analysis and our analysis in AdS is that in AdS, there is a term in $E(r)$ which grows quadratically, thereby dominating over the cubic term which is the leading contribution in Myers' analysis. Closer examination reveals that the quadratic term arises from the r dependence of g_{00} which enters into the Nambu-Born-Infeld action. g_{00} has non-trivial r dependence because the space-time is curved in response to the stress energy generated by the cosmological constant in the AdS space.

However, there is also stress energy associated with the field strength in the system considered by Myers. The stress energy due to the background field strength can certainly give rise to a non-trivial r dependence in the background metric. This is the effect of gravitational back reaction of the anti-symmetric form field strength background which was ignored in Myers' analysis. However, if the effect of such a back reaction enters at quadratic order, it can drastically alter the conclusion regarding the classical instability in the small r region.

We claim that this is indeed the case. A complete analysis for that issue in full generality is beyond the scope of this paper. Instead we shall demonstrate that point by considering an example which is sufficiently generic. We consider a version of Myers' mechanism where a D1-brane is blown up into a spherical D3-brane in the presence of RR 5-form field strength. The reason that this should be considered sufficiently generic is the fact that in string theory, RR 5-form background can only be created using a source that is available in the theory. The object which acts as a source for RR 5-form field strength is the D3-brane. D3-branes are especially convenient because they do not act as a source for the dilaton.[4]

Consider taking a large number of D3-branes oriented along the 0123 directions, distributed arbitrarily along the 456789 directions. Our ability to construct consistent backgrounds will be parameterized by the degree of freedom in distributing the D3-branes. The general form of backgrounds that can be generated this way takes the form [15]

$$\begin{aligned} ds^2 &= f^{-1/2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + f^{1/2}(dx_4^2 + dx_5^2 + dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2)\,, \\ F_{0123i} &= \partial_i f^{-1}\,, \end{aligned} \tag{A.2}$$

where $f(x_4, x_5, x_6, x_7, x_8, x_9)$ is an arbitrary harmonic function on the 456789-plane.

At some fixed point in the 456789 plane, the invariant field strength squared is given by

$$F^2 = \frac{(\partial_i f)^2}{f^{5/2}}\,. \tag{A.3}$$

Using only the fact that f is a harmonic function ($\nabla^2 f = 0$), one finds that at the neighborhood of that point

$$R_{ij} = -F^2 g_{ij}\,, \qquad R_{ab} = F^2 g_{ab}\,, \tag{A.4}$$

where i and j runs over 0123 and the direction of the gradient $\vec{\nabla} f$, whereas a and b runs over the rest of the directions. Therefore, at any point in the 456789 plane, the local geometry, to quadratic order in geodesic distance, is an $AdS_5 \times S^5$ with the radius of the order $R^2 = 1/F^2$. This in turn implies that there will always be a quadratic correction to the g_{00} component of the metric in the locally inertial frame

$$g_{00} \approx -(1 + F^2 r^2 + \cdots)\,. \tag{A.5}$$

This term will always give rise to a quadratically rising potential in $E(r)$

$$\begin{aligned} E(r) &= T_2 \left(\sqrt{-g_{00}\left(\frac{\alpha'^2 N^2}{4} + r^4\right)} - F r^3 \right) \\ &\approx T_2 \left(\frac{N\alpha'}{2} + N\alpha' F^2 r^2 - F r^3 + \cdots \right) \end{aligned} \tag{A.6}$$

which dominates over the r^3 term at small r. Because this is the leading effect at small r, it is inconsistent to ignore the effect of gravitational back reaction. When this effect is properly taken into account, there will never be a classical instability at $r = 0$ for a freely falling D1-brane. It should be emphasized however that this discussion is valid only locally. In general it might be that the minimum at $r = 0$ is only a local minimum and not a global minimum.

[4] This is not the most general possibility. For example, RR waves with no D-branes sources are excluded from this class of backgrounds.

B. Supersymmetry condition for branes in $AdS_5 \times S^5$

In this appendix, we will analyze the supersymmetric properties of the spherical branes in AdS_5 and S^5, as well as the instanton solutions describing the tunneling between the degenerate vacua. The strategy is to simply apply the supersymmetry condition [16, 17] locally and to count the supersymmetries that are left unbroken globally. Similar strategies have been applied to study the global supersymmetries of baryonic configurations in $AdS_5 \times S^5$ [18, 19]. The main complication arises from the fact that both the background and the brane are curved. Following [18], let us introduce

$$\epsilon^{\pm}(x) = \epsilon_L(x) \pm i\epsilon_R(x)\,, \tag{B.1}$$

where $\epsilon_{L,R}(x)$ are the left and right handed Majorana-Weyl Killing spinors of type-IIB supergravity on $AdS_5 \times S^5$, with positive spacetime chirality $\Gamma_{11} = +1$. The local supersymmetry condition for D3-branes under consideration takes the form

$$\Omega^{ijkl}(x)\Gamma_{ijkl}\epsilon^{\pm}(x) = \mp i\epsilon^{\pm}(x)\,, \tag{B.2}$$

where $\Omega^{ijkl}(x)$ is proportional to the volume element of the D3-brane. It is convenient to write the covariantly constant spinors $\epsilon^{\pm}(x)$ in the form

$$\epsilon^{\pm}(x) = S^{\pm}(x)\epsilon_0^{\pm} \tag{B.3}$$

so that the local supersymmetry condition reads

$$\Omega^{ijkl}(x)\Gamma_{ijkl}S^{\pm}(x)\epsilon_0^{\pm} = \mp iS^{\pm}(x)\epsilon_0^{\pm}\,. \tag{B.4}$$

The number of independent spinors $\epsilon_0^{\pm}$ satisfying the condition (B.4) for all x is the number of unbroken supersymmetries of the brane configuration. It should be emphasized that the condition on $\epsilon_0^{\pm}$ at different values of x is an overcomplete set, and that there exist any $\epsilon_0^{\pm}$ at all that satisfies this requirement is highly non-trivial.

This non-trivial condition is satisfied by the brane configurations illustrated in figure 2. The condition on $\epsilon_0^{\pm}$ simplifies to

$$(1 - \Gamma_\tau\Gamma_\phi)\epsilon_0^{\pm} = 0\,, \tag{B.5}$$

where Γ_τ and Γ_ϕ are the Γ matrices associated with the time direction and the direction of the orbit of the branes in S^5 respectively. This condition is exactly the same as that for massless particles in ten dimensions, and the same condition applies to all of the brane configurations illustrated in figure 2. In other words, these branes are indistinguishable at the level of supersymmetries, and they all belong to the same supermultiplet as that of the supergraviton.

Instanton solutions describing the tunneling between the spherical and the point like branes also preserve some fraction of supersymmetries. In addition to (B.5), the constraint on supersymmetires imposed by the instanton solution takes the form

$$\Gamma_{r\phi_1\phi_2\phi_3}\epsilon_0^{\pm} = \epsilon_0^{\pm}\,, \tag{B.6}$$

where Γ_r and $\Gamma_{\phi_1\phi_2\phi_3}$ are the Γ matrices associated with the radial and the three-sphere directions. Therefore, these instantons preserve one quarter of the supersymmetries.

In the remainder of this appendix, we will summarize the argument leading to the conclusion that the conditions for supersymmetry are given by (B.5) and (B.6). To proceed, we need an explicit expression for the Ω^{ijkl} and the $S^{\pm}(x)$. Let us begin by choosing an explicit coordinates for $AdS_5 \times S^5$. We will continue to use the metric (2.2) and parameterize the 3-sphere and the 5-sphere according to

$$d\Omega_3^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 \tag{B.7}$$

$$\begin{aligned} d\Omega_5^2 &= d\theta_{\bar 5}^2 + \sin^2\theta_{\bar 5} d\theta_{\bar 4}^2 + \sin^2\theta_{\bar 5}\sin^2\theta_{\bar 4} d\theta_{\bar 3}^2 + \sin^2\theta_{\bar 5}\sin^2\theta_{\bar 4}\sin^2\theta_{\bar 3} d\theta_{\bar 2}^2 + \\ &\quad + \sin^2\theta_{\bar 5}\sin^2\theta_{\bar 4}\sin^2\theta_{\bar 3}\sin^2\theta_{\bar 2} d\theta_{\bar 1}^2 \,. \end{aligned} \tag{B.8}$$

We will generally use barred indecies to refer to S^5 coordinates and unbarred indecies to refer to AdS_5 coordinates.

We will follow the Γ matrix conventions of [20]:

$$\Gamma_m = \sigma_2 \otimes \gamma_m^{AdS} \otimes 1_4 \,, \qquad \Gamma_{\bar m} = \sigma_1 \otimes 1_4 \otimes \gamma_{\bar m}^{S} \,, \qquad \gamma = \Gamma_{\bar 1\bar 2\bar 3\bar 4\bar 5} = \sigma_1 \otimes 1_4 \otimes 1_4 \,, \tag{B.9}$$

where $m = 0,1,2,3,4$ and $\bar m = \bar 1, \bar 2, \bar 3, \bar 4, \bar 5$. An explicit form of the Killing spinors in $AdS_5 \times S^5$ can be obtained by combining the results of [20] and [21]

$$S^{\pm}(x) = \sqrt{\sec\rho}\left(\cos\frac{\rho}{2} \pm i\hat x^{\alpha}\gamma\Gamma_{\alpha}\sin\frac{\rho}{2}\right) e^{\pm i\frac{\tau}{2}\gamma\Gamma_0} e^{\pm\frac{i}{2}\theta_{\bar 5}\gamma\Gamma_{\bar 5}} \prod_{m=4}^{1}\left(e^{-\frac{1}{2}\theta_{\bar m}\Gamma_{\bar m,\bar m+\bar 1}}\right), \tag{B.10}$$

where $\hat x_i$ are defined following [21]

$$\begin{aligned} \hat x_1 &= \sin\theta_1 \sin\theta_2 \sin\theta_3 \\ \hat x_2 &= \sin\theta_1 \sin\theta_2 \cos\theta_3 \\ \hat x_3 &= \sin\theta_1 \cos\theta_2 \\ \hat x_4 &= \cos\theta_1 \,. \end{aligned} \tag{B.11}$$

B.1 Supersymmetry of the spherical branes

We are now ready to analyze the unbroken supersymmetries of the spherical D3-branes in AdS_5. Let us take $\theta_{\bar 5}$ to be the direction of orbit along the S^5, more specifically the other angles $(\theta_{\bar 4}, \theta_{\bar 3}, \theta_{\bar 2})$ set to be zero, and $\theta_{\bar 1}$ to be 0 or π, to cover the orbit globally by two patches. Then the volume form of the spherical D3-brane takes the form of

$$\Omega = \frac{R^4(\sec^2\rho d\tau + \omega d\theta_{\bar 5})}{\sqrt{\sec^2\rho - \omega^2}} \wedge d\theta_3 \wedge d\theta_2 \wedge d\theta_1 \,. \tag{B.12}$$

Thus $\Omega^{ijkl}\Gamma_{ijkl}$ is given by

$$\Omega^{ijkl}\Gamma_{ijkl} = \frac{1}{\sqrt{\sec^2\rho - \omega^2}} \left(\sec\rho\Gamma_0 + \omega\Gamma_{\bar{5}}\right)\left(-\hat{x}_1\Gamma_{234} + \hat{x}_2\Gamma_{134} - \hat{x}_3\Gamma_{124} + \hat{x}_4\Gamma_{123}\right). \tag{B.13}$$

Now for the point-like solution at $\rho = 0$, $\omega = 1$ (figure 2.**I**), and on the $\theta_{\bar{1}} = 0$ branch, it is easy to show that (B.4) implies

$$(1 - \Gamma_0\Gamma_{\bar{5}})\epsilon_0^{\pm} = 0. \tag{B.14}$$

On the $\theta_{\bar{1}} = \pi$ branch, $\omega = -1$ since the orientation of the orbit is reversed.

For the finite size solution at $\tan\rho = \sqrt{L/N}$, $\omega = 1$ (figure 2.**III**), after straightforward manipulation of Γ matrices, (B.4) also reduces to (B.14). The same remark on the two branches of orbit is applicable in this case as well. Note that the condition $\omega = 1$ (or $\omega = -1$) is actually necessary for preserving the global supersymmetries.

The unbroken supersymmetry of spherical branes in S^5 (figure 2.**II**) can be analyzed in a similar manner. We will again take $\theta_{\bar{5}}$ to be the direction of orbit of the center of mass. Then the volume form takes the form of

$$\Omega = \frac{R^4(d\tau + \omega(1 - r^2/R^2)d\phi)}{\sqrt{1 - (1 - r^2/R^2)\omega^2}} \wedge d\theta_{\bar{3}} \wedge d\theta_{\bar{2}} \wedge d\theta_{\bar{1}}\,, \tag{B.15}$$

where $\tan\phi = \tan\theta_{\bar{5}}\cos\theta_{\bar{4}}$, $r = R\sin\theta_{\bar{5}}\sin\theta_{\bar{4}}$, and the brane is sitting at the origin $\rho = 0$ in AdS_5. Thus we have

$$\Omega^{ijkl}\Gamma_{ijkl} = \frac{1}{\sqrt{1 - (1 - r^2/R^2)\omega^2}} \left\{\Gamma_0 + \omega\left(\cos\theta_{\bar{4}}\Gamma_{\bar{5}} - \frac{r}{R\tan\theta_{\bar{5}}}\Gamma_{\bar{4}}\right)\right\}\Gamma_{\bar{3}\bar{2}\bar{1}}\,. \tag{B.16}$$

After some manipulation, once again (B.4) simplifies to

$$(1 - \Gamma_0\Gamma_{\bar{5}})\epsilon_0^{\pm} = 0\,. \tag{B.17}$$

B.2 Supersymmetry of the instantons

Now we proceed to the analysis of unbroken supersymmetries of instantons on the spherical D3-branes in AdS_5. The analysis goes through in much the same way as in the previous cases. The only difference comes from the time-dependence on the radial direction which will be reflected in the time-like direction of the worldvolume of the spherical D3-branes. As a result $\Omega^{ijkl}\Gamma_{ijkl}$ takes the form of

$$\Omega^{ijkl}\Gamma_{ijkl} = \frac{1}{\sqrt{\sec^2\rho - \dot{\rho}^2\sec^2\rho - \omega^2}} \left(\sec\rho\Gamma_0 + \dot{\rho}\sec\rho\hat{x}^{\alpha}\Gamma_{\alpha} + \omega\Gamma_{\bar{5}}\right)\left(-\hat{x}^{\beta}\Gamma_{\beta}\Gamma_{1234}\right). \tag{B.18}$$

After a little computation, one finds the global supersymmetry conditions to be

$$(1 - \Gamma_0\Gamma_{\bar{5}})\,\epsilon_0^{\pm} = 0\,, \tag{B.19}$$

$$\Gamma_{1234}\epsilon_0^{\pm} = \epsilon_0^{\pm}\,, \tag{B.20}$$

$$\omega \mp i\dot{\rho}\tan\rho - 1 = 0\,. \tag{B.21}$$

Using the relation between ω and L, it is easy to show that (B.21) is the instanton equation (in euclidean time)

$$\mp\dot{\rho} = \tan\rho \frac{\tan^2\rho - L/N}{\tan^4\rho + L/N} \tag{B.22}$$

whose solution is (3.4).

Similarly on the spherical D3-branes in S^5, $\Omega^{ijkl}\Gamma_{ijkl}$ is given by

$$\Omega^{ijkl}\Gamma_{ijkl} = \frac{1}{\sqrt{1 - \frac{(\dot{r}/R)^2}{1-(r/R)^2} - (1-(r/R)^2)\omega^2}} \left\{ \Gamma_0 + \frac{\dot{r}/R}{1-(r/R)^2} \Big(\cos\theta_{\bar{5}} \sin\theta_{\bar{4}} \Gamma_{\bar{5}} + \right.$$
$$\left. + \cos\theta_{\bar{4}}\Gamma_{\bar{4}} \Big) + \omega \left(\cos\theta_{\bar{4}}\Gamma_{\bar{5}} - \frac{r}{R\tan\theta_{\bar{5}}}\Gamma_{\bar{4}} \right) \right\} \Gamma_{\bar{3}\bar{2}\bar{1}} \,. \tag{B.23}$$

This time, the condition for preservation of global supersymmetry turns out to be

$$(1 - \Gamma_0\Gamma_{\bar{5}})\,\epsilon_0^{\pm} = 0\,, \tag{B.24}$$
$$\Gamma_{\bar{1}\bar{2}\bar{3}\bar{4}}\epsilon_0^{\pm} = \epsilon_0^{\pm}\,, \tag{B.25}$$
$$\omega \pm i\frac{r}{R}\frac{\dot{r}/R}{1-(r/R)^2} - 1 = 0\,. \tag{B.26}$$

Once again one can easily verify that the last condition (B.26) is precisely the instanton equation

$$\pm\frac{\dot{r}}{R} = \frac{r}{R}\left\{ \frac{N}{L}\left(\frac{r}{R}\right)^2 - 1 \right\} \tag{B.27}$$

whose solution is (3.6).

C. Generalizations to other AdS

In this article, we concentrated mainly on spherical branes in $AdS_5 \times S^5$. This can be generalized immediately to $AdS_7 \times S^4$ and $AdS_4 \times S^7$. Following the argument presented in section 3, one obtains an expression for the energy as a function of the angular momentum L and the radius ρ

- M2 in $AdS_4 \times S^7$

$$E(\rho, L) = N\left(\sec\rho \sqrt{\frac{L^2}{4N^2} + \tan^4\rho} - \tan^3\rho \right) \tag{C.1}$$

- M5 in $AdS_7 \times S^7$

$$E(\rho, L) = N\left(\sec\rho \sqrt{\frac{4L^2}{N^2} + \tan^{10}\rho} - \tan^6\rho \right). \tag{C.2}$$

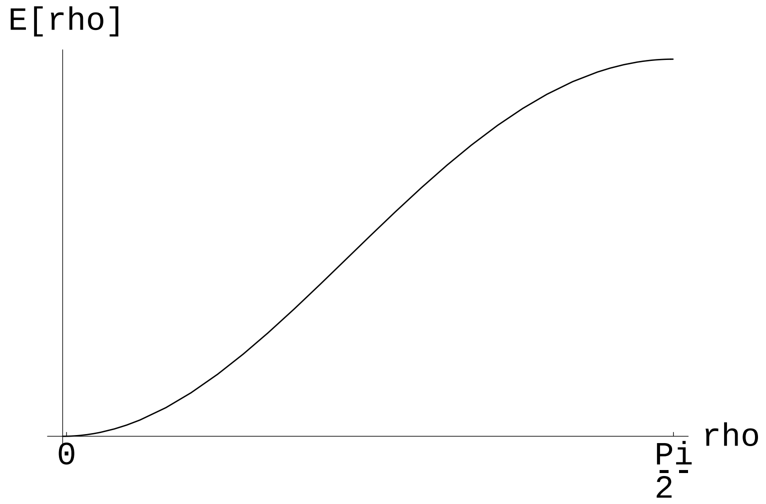

Figure 6: $E(\rho, L)$ for spherical D1-brane in $AdS_3 \times S^3$.

These potentials essentially behave like (3.2) illustrated in figure 1. These are the electric counterparts to the spherical magnetic dipoles in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ described in [5].

The case of $AdS_3 \times S^3$ is somewhat different. Consider a D1-string probe in the $AdS_3 \times S^3 \times T^4$ background with Q_1 and Q_5 units of electric and magnetic Ramond-Ramond 3-form fluxes, respectively. The potential energy then takes the form

$$E(\rho, L) = Q_5 \left(\sec\rho \sqrt{\left(\frac{L^2}{Q_5^2}\right) + \tan^2\rho} - \tan^2\rho \right) . \tag{C.3}$$

The potential has one global minimum at $\rho = 0$. There is an unstable stationary point at $\rho = \pi/2$ (see figure 6). This is precisely the long string of [11]. At the special value of angular momentum $L = Q_5$, the potential becomes completely flat and the long and the short strings become degenerate [11]. The fact that this happens at $L = Q_5$ rather than $L = Q_1 Q_5$ indicates that this effect is unrelated to the stringy exclusion principle of $AdS_3 \times S^3$.

Spherical D-strings in S^3 can also be analyzed in similar manner. The potential is found to take the form

$$E = \sqrt{\frac{Q_5^2 r^2}{R^4} + \frac{(L^2 - Q_5 r^2/R^2)^2}{R^2 - r^2}} , \tag{C.4}$$

which for $L = Q_5$ becomes flat and degenerate.

Note added. While this paper was in preparation, we learned that similar results are being considered by Grisaru, Myers, and Tafjord [22].

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